

ME 403

HOME WORK # 2

$$U = \int_0^{L/2} \frac{M^2}{2EI} dx + \int_{L/2}^L \frac{M^2}{2E(2I)} dx$$

$$U_1 = \int_0^{L/2} \frac{(FX)^2}{2EI} dx$$

$$= \frac{1}{3} \frac{F^2 x^3}{2EI} \Big|_0^{L/2} = \frac{1}{6} F^2 \left(\frac{L}{2}\right)^3$$

$$U_1 = \frac{1}{6} \frac{F^2}{EI} \left(\frac{L}{2}\right)^3 = \frac{1}{48} \frac{F^2 L^3}{EI}$$

$$U_2 = \int_0^{L/2} \frac{\left[F\left(\frac{L}{2} + x\right)\right]^2}{4EI} dx$$

$$= \frac{F^2}{4EI} \int_0^{L/2} \left(\frac{L^2}{4} + Lx + x^2\right) dx$$

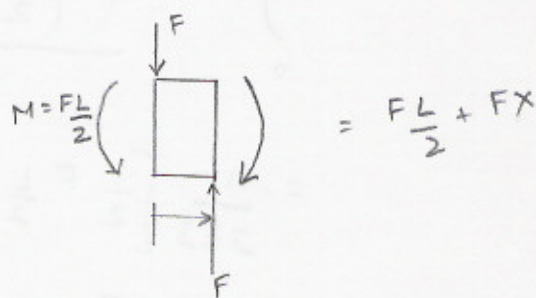
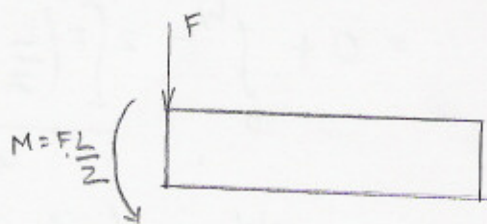
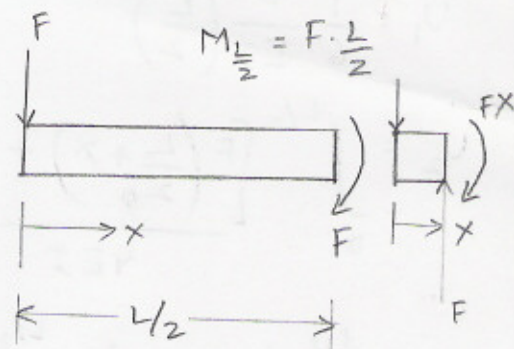
$$= \frac{F^2}{4EI} \left(\frac{L^2}{4}x + \frac{L}{2}x^2 + \frac{1}{3}x^3\right) \Big|_0^{L/2}$$

$$U_2 = \frac{F^2}{4EI} \left(\frac{L^3}{8} + \frac{L^3}{8} + \frac{L^3}{24}\right) = \frac{7}{9} \frac{F^2 L^3}{EI}$$

$$U = U_1 + U_2$$

$$= \frac{1}{48} \frac{F^2 L^3}{EI} + \frac{7}{9} \frac{F^2 L^3}{EI}$$

$$\frac{dU}{dF} = \frac{18}{96} \frac{FL^3}{EI} = \frac{3}{16} \frac{FL^3}{EI}$$



This is the total deflection

$$U_1 = \frac{1}{6} \frac{F^2}{EI} \left(\frac{L}{2}\right)^3$$

$$U_2 = \int_0^{L/2} \frac{\left[F\left(\frac{L}{2} + x\right) + Qx\right]^2}{4EI} dx$$

$$\frac{dU_1}{dQ} = \frac{dU_1}{dQ} + \frac{d}{dQ} \int_0^{L/2} \frac{\left[F\left(\frac{L}{2} + x\right) + Qx\right]^2}{4EI} dx$$

$$= 0 + \int_0^{L/2} \frac{2\left[F\left(\frac{L}{2} + x\right) + Qx\right]x}{4EI} dx$$

$$= \int_0^{L/2} \frac{2\left(F\left(\frac{L}{2} + x\right)x + 2Qx^2\right)}{4EI} dx$$

$$= \frac{1}{2EI} \left[\frac{1}{2} F \frac{L}{2} x^2 + \frac{1}{3} F x^3 + \frac{1}{3} Q x^3 \right]_0^{L/2}$$

$$= \frac{1}{2EI} \left[\frac{1}{16} FL^3 + \frac{1}{24} FL^3 + \frac{1}{24} QL^3 \right]$$

Setting $Q=0$,

$$\frac{dU}{dQ} = \frac{1}{2EI} \left[\frac{1}{16} FL^3 + \frac{1}{24} FL^3 \right]$$

$$\boxed{\frac{dU}{dQ} = \frac{5}{96} \frac{FL^3}{EI}}$$

This is the deflection at A

